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CP Violation and New Physics*

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We describe some of the extensions of the SM, including models with spontaneous CP violation, where New Physics relevant for CP violation may arise. It is emphasized that the SM predicts a series of exact relations among various measurable quantities, such as moduli of CKM matrix elements and rephasing invariant phases. These exact relations provide a stringent test of the SM, with the potential to reveal New Physics.

1 Introduction

The study of CP violation in its multiple aspects is likely to continue playing a crucial role in testing the Standard Model(SM) and in searching for New Physics. So far, all experimental data on flavour physics and CP violation [1] are in agreement with the SM and its Kobayashi-Maskawa (KM) mechanism [2]. This agreement is impressive, since one has to accommodate a large number of data with only a few parameters. The Cabibbo, Kobayashi and Maskawa (CKM) matrix is characterized by four parameters which one can choose to be the three angles θ_i and the phase δ of the standard parametrization [3]. The values of s_1, s_2 and s_3 ($s_i = \sin\theta_i$) can be determined by the experimental value of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$. Once these parameters are fixed, one has to fit, using only the phase δ, a large amount of data, such as ε_K, ε'/ε, $\sin(2\beta)$, ΔM_{B_d}, ΔM_{B_s}. It is remarkable that these five experimental quantities can be fitted with only one parameter, namely the KM phase δ.

In spite of this success of the SM, the search for New Physics through the study of CP violation phenomena is well motivated by various reasons, such as:

i) CP violation is closely related to the least understood aspects of the SM, namely the Higgs sector and the structure of Yukawa couplings.

ii) Almost any extension of the SM has new sources of CP violation.

iii) CP violation is one of the crucial ingredients needed to generate the Baryon Asymmetry of the Universe (BAU). It has been established that the strength of CP violation in the SM is not sufficient to generate the observed BAU. Therefore, in all successful baryogenesis scenarios, including baryogenesis through leptogenesis [4] , new sources of CP violation are present.

iv) Although CP violation can be incorporated in the SM through the introduction of complex Yukawa couplings, one would like to have a deeper understanding of the origin of CP violation. Such an understanding will certainly require a framework of physics beyond the SM. One may ask, for example, the question whether there are any connections among the various possible manifestations of CP violation, namely those in the quark sector, and in the leptonic sector. In particular one may ask whether there is any relation between leptonic CP violation observable in neutrino oscillations and CP violation needed for leptogenesis [5], [6], [7]. Or one may also wonder whether all manifestations of CP violation have a common origin [8].

In most of the extensions of the SM, it is necessary to control the new sources of CP violation in order to conform to the experimental value of ε_K, as well as to the experimental limits on the electric dipole moments of the neutron and the electron. A notable example is the supersymmetric extension of the SM, where in general a very large number of new phases arise, leading to the so called supersymmetric CP problem, which can be solved by either assuming that the new phases are small or by having an alternative suppression mechanism [9].

We will not present a general discussion of models of CP violation since it is beyond the scope of this contribution. Instead, we will divide models of CP violation into two broad classes, based on the nature of CP breaking, which may be spontaneous or explicit. One of the motivations for having spontaneous CP violation, as emphasized by Lee [10] in his pioneering work is puting the breaking of CP on the same footing as the breaking of gauge symmetry, which is spontaneously broken. Another motivation has to do with the fact that spontaneous CP breaking provides an alternative solution to the strong CP problem [11], [12], [13] [14] (apart from the Peccei Quinn solution [15]) in models where $\overline{\theta}$ naturally vanishes at tree level and it is calculable in higher orders. Furthermore, it was shown some time ago that CP can be spontaneously broken in string theory [16] and more recently it has been pointed out that in some string theory compactifications, CP is an exact gauge symmetry and thus its breaking has to be spontaneous [17].

*The analysis presented in section 3 was done in collaboration with F. J. Botella, M. Nebot and M. N. Rebelo in Ref. [44]

2 The Breaking of CP: Explicit or Spontaneous?

Indeed, one of the basic questions one may ask about CP is whether it is explicitly broken at the Lagrangian level or, on the contrary, it is a good symmetry of the Lagrangian, spontaneously broken by the vacuum. It is remarkable that two of the most interesting models of CP violation, suggested in the early days of gauge theories were published in the same year (1973) and belong to each one of the above categories. We are refering to the Two-Higgs-Doublet-Model (THDM) suggested by Lee, where CP is spontaneously broken and the celebrated KM model [2] where CP is explicitly broken at the Lagrangian level, through the introduction of complex Yukawa couplings. Next we consider some of the simplest extensions of the SM which allow for spontaneous CP violation.

2.1 The Lee Model

It can be readily shown that in the SM with only one Higgs doublet, CP cannot be spontaneously broken. Lee has proposed a minimal extension of the SM where spontaneous CP violation (SCPV) can be achieved, through the introduction of two Higgs doublets. Due to the presence in the Higgs potential of terms of the form $\Phi_1^\dagger\Phi_2\Phi_1^\dagger\Phi_2$, $\Phi_1^\dagger\Phi_2\Phi_1^\dagger\Phi_1$, $\Phi_2^\dagger\Phi_2\Phi_1^\dagger\Phi_2$ the potencial is sensitive to the relative phase between the vevs of the two neutral Higgs fields, $<\phi_j^0>= v_j \exp(i\theta_j)$. There is a region of the parameters of the Higgs potential for which its minimum corresponds to a non vanishing $\theta = (\theta_2 - \theta_1)$. This leads in general to spontaneous CP violation. At the time Lee suggested this model, only two (incomplete) generations were known. In this case, the only source of CP violation was Higgs exchange. If one considers the THDM with SCPV in the framework of the 3-fermion-generations SM, a non-trivial KM phase is generated in the CKM matrix, in spite of the fact that Yukawa couplings are real. This can be readily verified, by noting that the quark mass matrices for the down and up quarks have the form:

$$\begin{aligned} M_d &= \frac{1}{\sqrt{2}}\left[v_1 e^{i\theta_1} Y_1^d + v_2 e^{i\theta_2} Y_2^d\right] \\ M_u &= \frac{1}{\sqrt{2}}\left[v_1 e^{-i\theta_1} Y_1^u + v_2 e^{-i\theta_2} Y_2^u\right] \end{aligned} \tag{1}$$

where Y_k^d, Y_k^u stand for the Yukawa coupling matrices. One obtains for the hermitian quark mass matrices:

$$\begin{aligned} H_d &\equiv M_d M_d^\dagger = \tfrac{1}{2}[v_1^2 Y_1^d Y_1^{dT} + v_2^2 Y_2^d Y_2^{dT} \\ &\quad +v_1 v_2 (Y_1^d Y_2^{dT} + Y_2^d Y_1^{dT}) \cos\theta \\ &\quad -i v_1 v_2 (Y_1^d Y_2^{dT} - Y_2^d Y_1^{dT}) \sin\theta] \qquad (2) \\ H_u &\equiv M_u M_u^\dagger = \tfrac{1}{2}[v_1^2 Y_1^u Y_1^{uT} + v_2^2 Y_2^u Y_2^{uT} \\ &\quad +v_1 v_2 (Y_1^u Y_2^{uT} + Y_2^u Y_1^{uT}) \cos\theta \\ &\quad +i v_1 v_2 (Y_1^u Y_2^{uT} - Y_2^u Y_1^{uT}) \sin\theta] \qquad (3) \end{aligned}$$

Although there is only one physical phase, namely $\theta = (\theta_2 - \theta_1)$, it is clear from Eqs. (2) and (3) that due to the arbitrariness of Y_k^d, Y_k^u, the matrices H_u, H_d are arbitrary hermitian matrices. As a result, there will be in general a non-trivial CP violating phase in the CKM matrix. For any specific choice of Y_k^d, Y_k^u this can be explicitly verified by computing the invariant [18] quantity $T \equiv tr[H_u, H_d]^3$. For three fermion generations the non-vanishing of this weak-basis invariant is a necessary and sufficient condition for having CP violation mediated by charged weak-interactions.

From the above discussion, one concludes that in the three fermion generations version of the Lee model, one has two sources of CP violation, namely:

i) The usual KM mechanism contributing to CP violation in decay amplitudes as well as to $K^0 - \overline{K}^0$, $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixings through the usual box diagrams.

ii) Flavour-changing neutral Higgs mediated interactions giving additional tree level contributions to the neutral meson mixings mentioned in i).

The above generalization of the Lee model to three fermion generations illustrates a very common situation, where one has the KM mechanism, together with other sources of CP violation.

One of the potential drawbacks of the Lee model, is the fact that the existence of Higgs mediated flavour-changing neutral currents(FCNC) at tree level requires very heavy neutral scalars, of the order of a few Tev, unless there is some suppression mechanism [19]. It has been shown that in the framework of two-Higgs-doublets models, the introduction of appropriate discrete symmetries [20] leads to the suppression of the FCNC vertex between, for example, two down-type quarks i and j by products of the CKM matrix elements of the type $V_{\alpha i}^* V_{\alpha j}$, where α denotes one of the up-type quarks. In the case $\alpha = t$, $i = d$, and $j = s$ this suppression is quite strong, and in that class of models neutral Higgs may be relatively light (e.g. 100-200 GeV), even in the presence of Higgs mediated FCNC.

2.2 Multi-Higgs Models with Natural Flavour Conservation

One may, of course, eliminate altogether FCNC in the two-Higgs-doublets models by implementing Natural Flavour Conservation (NFC) in the Higgs sector through a Z_2 discrete symmetry [21]. However, in this case, the structure of the Higgs potential is such that no spontaneous CP violation can be achieved, unless the discrete symmetry is

softly broken [22]. If one insists on NFC, then a minimum number of three Higgs doublets are necessary in order to achieve spontaneous CP violation [23]. This class of models with NFC, three Higgs doublets and SCPV has the special feature that the CKM matrix is real [24] and CP violation arises exclusively through Higgs exchange [25]. The essential reason why the CKM is real in this case has to do with the fact that the Z_2 symmetry constrains d_R to couple to only one of the Higgs doublets (and similarly for u_R). In this case, any phase can be rotated away from the quark mass matrices, through a redefinition of the righthanded quark fields. In the version of the three Higgs-doublet model, with explicit CP violation [26] and in the presence of three fermion generations, one has again two sources of CP violation, namely the KM mechanism and Higgs exchange.

2.3 SCPV in Supersymmetric Extensions of the SM

The Minimal Supersymmetric Standard Model (MSSM) has two Higgs doublets and therefore it is a natural candidate to achieve SCPV. However, it is not possible to obtain SCPV in the MSSM at tree level, due essentially to the fact that SUSY does not allow some of the couplings which are present in the general THDM. Since SUSY has to be softly broken, radiative corrections can induce new CP violating operators which could induce CP breaking [27]. However, the possibility that radiative corrections can cause SCPV, requires the existence of a light scalar [28] which is excluded by LEP. It has been shown that one may achieve SCPV in the Next-to-Minimal-Supersymmetric-Standard-Model (NMSSM) [29], [30], [31] where a singlet superfield is added to the Higgs sector. In this case, the CKM matrix is real [24], essentially due to the same reason explained above for the 3-Higgs doublet model with SCPV and NFC. In the NMSSM with SCPV all couplings are real, so that CP is a good symmetry of the Lagrangian. However the physical relative phases of the Higgs doublet and singlet enter in the chargino and neutralino mass matrices as well as in some vertices. As a result, it has been shown that chargino box diagrams can generate [31] the observed experimetal value of ε_K. As far as ε'/ε, and $a_{J/\psi K_s}$, it has been pointed out that SUSY contributions in these models can saturate the experimental values [29] [32], provided there is maximal LR squark mixing.

2.4 Spontaneous CP Violation Generated at a High Energy Scale

If one maintains the fermion spectrum of the SM, the THDM suggested by Lee has the simplest Higgs structure needed to generate spontaneous CP breaking capable of accounting for the experimentally observed CP violation. However, it is possible to generate relevant spontaneous CP violation with only one Higgs doublet ϕ and one complex scalar singlet S, provided that one also introduces at least one singlet charge $-\frac{1}{3}$ vectorial quark D^0. The scalar potential will contain terms in ϕ and S with no phase dependence, together with terms of the form $(\mu^2 + \lambda_1 S^*S + \lambda_2 \phi^\dagger\phi)(S^2 + S^{*2}) + \lambda_3(S^4 + S^{*4})$ which, in general, lead to the spontaneous breaking of T and CP invariance [33] with ϕ and S acquiring vacuum expectation values (vevs) of the form:

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}, \quad \langle S \rangle = \frac{V \exp(i\alpha)}{\sqrt{2}} \tag{4}$$

In this class of models the presence of the vector-like quark D^0 plays a crucial rôle, since it is through the couplings $(f_q S + f_q{}' S^*)\overline{D_L^0} d_R^0$ that the phase α appears in the effective mass matrix for the down standard-like quarks. It can be shown that the phase δ_{KM}, generated through spontaneous CP violation is not suppressed by factors of $\frac{v}{V}$. For very large V (e.g. $V \sim M_{GUT} \sim 10^{15}$ Gev), δ_{KM} is the only leftover effect at low energies, from spontaneous CP breaking at high energies. For not so large a value of V (e.g., V of the order of a few Tev) the appearance of significant flavour changing neutral currents (FCNC) in the down quark sector leads to new contributions to $B_d - \overline{B_d}$ and $B_s - \overline{B_s}$ mixing which can alter [34] some of the predictions of the SM for CP asymmetries in B meson decays. These FCNC are closely related to the non-unitarity of the 3×3 CKM matrix, with both effects suppressed by powers of $\frac{v}{V}$.

This class of models has been extended to the leptonic sector where the role of vector-like quarks is played by the righthanded neutrinos. It was pointed out [8] that in such a framework, all manifestations of CP violation may have a common origin. In particular, the phase α defined in Eq.(4) generates CP violation in the quark sector, in the leptonic sector at low energies (measurable for example in neutrino oscillations), as well as CP violation required by leptogenesis.

3 Precision Tests of the SM and the Search for New Physics

From the discussion in the previous section it should be clear that when one considers extensions of the SM, the most common situation is having the usual KM mechanism, together with other sources of CP violation. This is of course the case when one assumes that CP is explicitly broken at the Lagrangian level. What is remarkable, is the fact that it is also true for some of the models of SCPV, where all the couplings of the Lagrangian are real.

In our analysis, we will assume that the tree level weak decays are dominated by the SM W-exchange diagrams, thus implying that the extraction of $|V_{us}|$, $|V_{ub}|$ and $|V_{cb}|$ from experiment continues to be valid even in the presence of New Physics (NP). We will allow for contributions from NP in processes like $B_d^0 - \overline{B}_d^0$ mixing and $B_s^0 - \overline{B}_s^0$ mixing, as well as in penguin diagrams. Since the SM contributes to these processes only at loop level, the effects of NP are

more likely to be detectable. Examples of processes which are sensitive to NP, are the CP asymmetries corresponding to the decays ${B^0}_d \to J/\Psi K_s$ and ${B^0}_d \to \pi^+\pi^-$ which are affected by NP contributions to $B^0_d - \overline{B}^0_d$ mixing. Significant contributions to $B^0_d - \overline{B}^0_d$ and $B^0_s - \overline{B}^0_s$ mixing can arise in many of the extensions of the SM, such as models with vector-like quarks [34] and supersymmetric extensions of the SM [35]. Vector-like quarks naturally arise in theories with large extra-dimensions [36], as well as in some grand-unified theories like E_6. As previously mentioned, the presence of vector-like quarks leads to a small deviation of 3×3 unitarity of V_{CKM} which in turn leads to Z-mediated new contributions to $B^0_d - \overline{B}^0_d$ and $B^0_s - \overline{B}^0_s$ mixings. In the minimal Supersymmetric Standard Model (MSSM) the size of SUSY contributions to $B^0_d - \overline{B}^0_d$ and $B^0_s - \overline{B}^0_s$ mixing crucially depends on the choice of soft-breaking terms, but there is a wide range of the parameter space where SUSY contributions can be significant. Recently, it has been pointed out [37] that in the context of SUSY SO(10), there is an interesting connection between the observed large mixing in atmospheric neutrinos and the size of the SUSY contribution to $B^0_s - \overline{B}^0_s$ mixing, which is expected to be large in this class of models.

The standard way of testing the compatibility of the SM with the existing data consists of adopting the Wolfenstein parametrization [38] and plotting in the ρ, η plane the constraints derived from various experimental inputs, like the value of ε_K, the size of $|V_{ub}| / |V_{cb}|$, the value of $a_{J/\psi K_s}$, as well as the strength of $B^0_d - \overline{B}^0_d$ and $B^0_s - \overline{B}^0_s$ mixings. The challenge for the SM is then to find a region in the ρ, η plane where all the constraints are simultaneously satisfied. A complementary way of testing the SM, consists of using exact relations connecting measurable quantities, namely moduli of V_{CKM} and the arguments of rephasing invariant quartets. These relations can be derived in the framework of the SM and follow from the implicit assumption of unitarity of V_{CKM}. They have the interesting feature of being independent of any particular parametrization of the quark mixing matrix.

3.1 Choice of Rephasing Invariant Phases

Using the freedom to rephase quark fields, it can be readily shown that the 3×3 sector of a CKM matrix of arbitrary size contains only four independent rephasing invariant phases. It is convenient to make the following choice:

$$\begin{aligned}
\gamma &\equiv \arg(-V_{ud}V_{cb}V^*_{ub}V^*_{cd}) = \arg\left(-\tfrac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right) \\
\beta &\equiv \arg(-V_{cd}V_{tb}V^*_{cb}V^*_{td}) = \arg\left(-\tfrac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right) \\
\chi &\equiv \arg(-V_{cb}V_{ts}V^*_{cs}V^*_{tb}) = \arg\left(-\tfrac{V_{cb}V^*_{cs}}{V_{tb}V^*_{ts}}\right) \\
\chi' &\equiv \arg(-V_{us}V_{cd}V^*_{ud}V^*_{cs}) = \arg\left(-\tfrac{V_{us}V^*_{ud}}{V_{cs}V^*_{cd}}\right)
\end{aligned} \tag{5}$$

Furthermore, in order to fix the invariant phases entering in B^0 CP asymmetries, it is useful to adopt the following phase convention [39]:

$$\arg(V) = \begin{pmatrix} 0 & \chi' & -\gamma \\ \pi & 0 & 0 \\ -\beta & \pi+\chi & 0 \end{pmatrix} \tag{6}$$

Through the measurement of CP asymmetries, one can obtain the phases of the rephasing invariant quantities:

$$\begin{aligned}
\lambda^{(q)}_f &= \left(\frac{q_{B_q}}{p_{B_q}}\right)\left(\frac{A\left(\overline{B}^0_q \to f\right)}{A\left(B^0_q \to f\right)}\right); \\
\lambda^{(q)}_{\overline{f}} &= \left(\frac{q_{B_q}}{p_{B_q}}\right)\left(\frac{A\left(\overline{B}^0_q \to \overline{f}\right)}{A\left(B^0_q \to \overline{f}\right)}\right)
\end{aligned} \tag{7}$$

The first factor in $\lambda^{(q)}_f$ is due to mixing and its phase equals (-2β) and 2χ for B_d and B_s, respectively. Let us consider the general case where New Physics (NP) also contributes to the mixing. It is convenient to parametrize the NP contributions in the following way:

$$\begin{aligned}
M^{(q)}_{12} &= \left(M^{(q)}_{12}\right)^{SM} r^2_q e^{-2i\phi_q} \Rightarrow \\
\Delta M_{B_q} &= \left(\Delta M_{B_q}\right)^{SM} r^2_q
\end{aligned} \tag{8}$$

$$\frac{q_{B_q}}{p_{B_q}} = \exp\left(i \arg\left(M^{(q)}_{12}\right)^*\right) = \left(\frac{q_{B_q}}{p_{B_q}}\right)^{SM} e^{2i\phi_q} \tag{9}$$

In the presence of NP, the phases from mixing become $2(-\beta+\phi_d) \equiv -2\overline{\beta}$ and $2(\chi+\phi_s) \equiv 2\overline{\chi}$ for B_d and B_s decays, respectively. It is clear that $r_q \neq 1$ and/or $\phi_q \neq 0$ would signal the presence of NP. It is not easy to separate β from a possible NP contribution (ϕ_d) in B^0_d decays like ${B^0}_d \to J/\Psi K_s$. This renders specially important the measurement of γ, which does not suffer from contamination of NP in the mixing. Note that γ can be either directly measured [40] or obtained through the knowledge of the asymmetries $a_{J/\Psi K_s} = \mathrm{Im}\left(\lambda^{(d)}_{J/\Psi K_s}\right)$, $a_{\pi^+\pi^-} = \mathrm{Im}\left(\lambda^{(q)}_{\pi^+\pi^-}\right)$. Indeed the phase ϕ_d cancels in the sum $\overline{\alpha} + \overline{\beta} = (\pi - \gamma - \beta + \phi_d) + (\beta - \phi_d)$ and one has:

$$\gamma = \pi - \frac{1}{2}\left[\arcsin a_{J/\Psi K_s} + \arcsin a_{\pi^+\pi^-}\right] \tag{10}$$

Note that we are using $a_{\pi^+\pi^-} = \sin(2\overline{\alpha})$ that can be extracted from the experimental asymmetry through various different approaches [41]. Once γ is known, β can be readily obtained, using unitarity and the knowledge of $|V_{ub}|$, $|V_{us}|$, $|V_{cb}|$. The knowledge of β, together with $a_{J/\Psi K_s}$ leads then to the determination of ϕ_d. Of course, this evaluation of ϕ_d will be restricted by the precision on $|V_{ub}|$, since $|V_{us}|$, $|V_{cb}|$

are extracted from experiment with good accuracy. Similar considerations apply to the extraction of r_d, r_s or r_d/r_s from ΔM_{B_d} and ΔM_{B_s} where $\left|V^*_{td}V_{tb}\right|$, $\left|V^*_{ts}V_{tb}\right|$ or its ratio, have to be reconstructed previously using unitarity.

3.2 Exact Relations

Using orthogonality of different rows and different columns of V_{CKM}, one can obtain various exact relations involving moduli and rephasing invariant phases, such as:

$$\sin\chi = \frac{|V_{td}|\,|V_{cd}|}{|V_{ts}|\,|V_{cs}|}\sin\beta \tag{11}$$

$$|V_{ub}| = \frac{|V_{cd}|\,|V_{cb}|}{|V_{ud}|}\frac{\sin\beta}{\sin(\gamma+\beta)} \tag{12}$$

$$\sin\chi = \frac{|V_{us}|\,|V_{cd}|\,|V_{cb}|}{|V_{ts}|\,|V_{tb}|\,|V_{ud}|}\frac{\sin\beta\sin(\gamma+\chi')}{\sin(\gamma+\beta)} \tag{13}$$

$$\frac{\sin\chi}{\sin(\gamma+\chi')} = \frac{|V_{us}|\,|V_{ub}|}{|V_{ts}|\,|V_{tb}|} \tag{14}$$

$$|V_{td}| = \frac{|V_{cd}|\,|V_{cb}|}{|V_{tb}|}\frac{\sin\gamma}{\sin(\gamma+\beta)} \tag{15}$$

Since the above formulae have the potential of providing precise tests of the SM, we have opted for writing exact relations. However, it is obvious that given the experimental knowledge on the size of the various moduli of the CKM matrix elements, some of the above relations can be, to an excellent approximation, substituted by simpler ones. For example, Eq.(13) is the exact version of the Aleksan-London-Kayser relation [42], the importance of which has been emphasized by Silva and Wolfenstein [43] :

$$\sin\chi \simeq \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{\sin\beta\sin\gamma}{\sin(\gamma+\beta)} \tag{16}$$

Similarly Eq.(11) can be well approximated by:

$$\sin\chi \simeq r\frac{|V_{us}|}{|V_{ud}|}\sin\beta \tag{17}$$

while Eqs.(15) and (14) lead, respectively to:

$$r \simeq |V_{us}|\frac{\sin\gamma}{\sin(\gamma+\beta)} \tag{18}$$

$$\sin\chi \simeq \frac{|V_{us}|\,|V_{ub}|}{|V_{cb}|}\sin\gamma \tag{19}$$

It is worthwhile to illustrate how these relations can be used to test the SM:

(i) Eq.(11) and its approximate form Eq.(17) would provide an excellent test of the SM, once χ, r and β are measured. Note that the theoretical errors in extracting $r \equiv |V_{td}|\,/\,|V_{ts}|$ from $B^0_d-\overline{B}^0_d$ and $B^0_s-\overline{B}^0_s$ mixings are much smaller than those present in the extraction of $|V_{td}|$, $|V_{ts}|$.

(ii) Eq.(19) has the important feature of only involving quantities which are not sensitive to the possible presence of New Physics in $B^0_d-\overline{B}^0_d$ mixing. It has, of course, the disadvantage of requiring the knowledge of $|V_{ub}|$ with significant precision, in order to be a precise test of the SM.

(iii) Eq.(18) gives, to an excellent approximation, r in terms of γ and β. This relation will provide an important test of the SM once r, γ and β are measured. Note that in the SM, one knows that r is of order $|V_{us}|$, the importance of Eq.(18) is that it provides the constant of proportionality.

In the context of the SM the above formulae can also be very useful for a precise determination of V_{CKM} from input data: for example, if β and γ are measured with sufficient accuracy, one can use Eqs.(12), (15) to determine $|V_{ub}|$, $|V_{td}|$. One can thus reconstruct the full CKM matrix, using $|V_{us}|$, $|V_{cb}|$, β and γ as input parameters. Furthermore we can also predict the SM value for $\sin 2\chi$ and $\sin\chi'$.

The above expressions can also be used to detect, in a quantitative way, the presence of New Physics in $B^0_d-\overline{B}^0_d$ and or $B^0_s-\overline{B}^0_s$ mixings. For example, Eq.(12), we see that this unitarity relation can only be affected by the presence of ϕ_d, therefore this equation allows for a clean extraction of ϕ_d. By writing Eq.(12) in terms of $\overline{\beta}$ and ϕ_d (note that $Im\left(\lambda^{(d)}_{J/\Psi K_s}\right) = \sin\left(2\overline{\beta}\right)$) we get

$$\tan(\phi_d) = \frac{R_u\sin\left(\gamma+\overline{\beta}\right)-\sin\left(\overline{\beta}\right)}{\cos\left(\overline{\beta}\right)-R_u\cos\left(\gamma+\overline{\beta}\right)} \tag{20}$$

with

$$R_u = \frac{|V_{ud}|\,|V_{ub}|}{|V_{cd}|\,|V_{cb}|} \tag{21}$$

From Eq.(20), we can find out the bounds that can be reached for ϕ_d, once we have a direct measurement of γ. To illustrate the usefulness of Eqs.(12) and (20) one can consider examples of different sets of assumed data which hopefully will be available in the near future. For definiteness let us consider the most optimistic scenario, where NP is discovered, corresponding to the following example [2]:

$$\begin{aligned} |V_{us}| &= 0.221 \pm 0.002 \\ |V_{cb}| &= 0.0417 \pm 0.0010 \\ |V_{ub}| &= (4.05 \pm 0.21) \times 10^{-3} \\ \overline{\beta} = (30.0 \pm 0.3)^{\,o} \gamma &= (20 \pm 5)^{\,o} \end{aligned} \tag{22}$$

the resulting ϕ_d distribution is presented in Fig.1 corresponding to $\phi_d = (-16.3 \pm 3.2)^{\,o}$ In this case, one would

[2] For more examples see Ref [44]

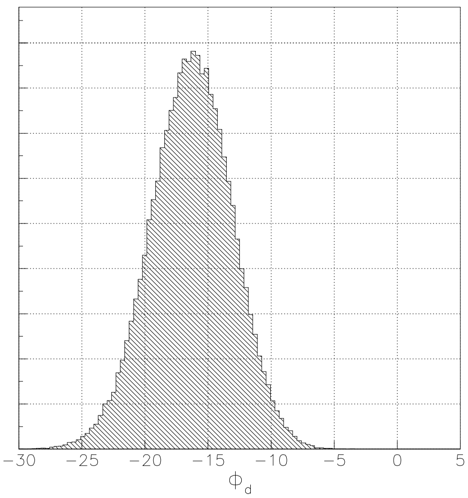

Figure 1. The ϕ_d distribution in degrees corresponding to an example where New Physics is clearly detected.

have a clear indication of NP in the phase of $B_d^0 - \overline{B}_d^0$ mixing. Note that for this choice of γ the value of ε_K would not be saturated by the SM contribution. Therefore, in this example one would conclude that NP also contributes to ε_K.

4 Conclusions

We have described the main features of CP violation in a variety of models beyond the SM, emphasizing that the most common situation is having the KM mechanism together with some extra sources of CP violation. Often this New Physics gives additional contributions to $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixings which can affect the predictions for the various CP asymmetries. Furthermore, we have pointed out that the SM predicts a series of exact relations connecting measurable quantities like moduli and rephasing invariant quartets of the CKM matrix which provide a stringent test of the SM, with the potential of revealing New Physics. This is specially true if, on the one hand, γ, x_s and eventually χ are measured in the present or future B factories and, on the other hand, there is a significant decrease in the theoretical uncertainties in the evaluation of the relevant hadronic matrix elements. These tests may complement the standard analysis in the ρ, η plane.

In the search for New Physics through CP violation, the first step is, of course, to find a clear deviation from the predictions of the SM for flavour physics and CP violation. If the need for New Physics is established through, for example, the appearance of new contributions to $B_d^0 - \overline{B}_d^0$ and/or $B_s^0 - \overline{B}_s^0$ mixings, a much more difficult task will be to differentiate among the various models where such new contributions may arise.

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